

Laissez-Faire Banking and Circulating Media of Exchange

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A model with private information that supports conventional arguments for a government monopoly in supplying circulating media of exchange is constructed. The model also yields rate-of-return and velocity predictions which are consistent with observations from free banking regimes and fiat money regimes. In a laissez-faire banking equilibrium, fiat money is (essentially) not valued, and the resulting allocation is not Pareto optimal. However, if private agents are restricted from issuing circulating notes, there exists an equilibrium with valued fiat money that Pareto dominates the laissez-faire equilibrium and is Pareto optimal within a restricted class of allocations. *Journal of Economic Literature* Classification Numbers: 020, 310. © 1992 Academic Press, Inc.

1. INTRODUCTION

In this paper a model that supports the conventional wisdom that it is desirable to have a government monopoly in supplying circulating media of exchange is constructed. This model also yields predictions consistent with what is observed in laissez-faire regimes with unrestricted private note issue and with what is observed in fiat money regimes. In particular, (1) circulating media of exchange are dominated in rate of return by other

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assets; (2) a laissez-faire equilibrium may have the property that fraudulent note issue is nonexistent, in spite of the fact that there is a *potential* for such practices; (3) bank deposit liabilities coexist in equilibrium with valued fiat money (in a fiat money regime) or with circulating notes (in a laissez-faire regime); (4) assets yielding relatively high (low) long-term rates of return have relatively low (high) transactions velocities.

The view that there is a legitimate basis for a government monopoly in supplying circulating media of exchange has been widely held in monetary economics. Milton Friedman, in outlining the drawbacks of laissez-faire monetary arrangements, argued that

. . . the [private] contracts in question are peculiarly difficult to enforce and fraud particularly difficult to prevent. The very performance of its central function requires money to be generally acceptable and to pass from hand to hand . . . in fraud as in other activities, opportunities for profit are not likely to go unexploited. (Friedman 1960, p. 6)

Thus, the private information aspects inherent in monetary exchange imply, according to this view, that an arrangement with unfettered private intermediation can be dominated by another arrangement with a government-supplied, universally recognizable medium of exchange and legal restrictions on private intermediaries.

There is a growing literature that puts laissez-faire monetary arrangements in a much more favorable light, thus challenging the conventional wisdom of Friedman and others. For example, Rolnick and Weber (1983, 1984) argue that the U.S. free banking era (1837–63), usually characterized as a laissez-faire banking regime, was much less chaotic than once thought. In particular, fraudulent banking practices, counterfeiting, and below-par redemption of free bank notes appear to have been the exception rather than the rule in most states during this period. The nineteenth century Scottish free banking system seems to have functioned with even fewer of the perceived problems of the U.S. free banks (see White, 1984). In informal theoretical reasoning, Hayek (1978) argues that efficiency gains would result from a move from current regimes to ones with unfettered financial arrangements, and Fama (1980, p. 47) argues that “. . . there is nothing in the economics of this [the banking] sector that makes it a special candidate for government control.” More formally, in Sargent and Wallace (1982), a Pareto optimal equilibrium allocation exists without restrictions on financial intermediation. In their model, if there are restrictions that effectively prohibit private intermediaries from issuing close substitutes for fiat money, then an equilibrium is not Pareto optimal.

In the model constructed here, there are overlapping generations of consumers, and each young agent has uncertain preferences over future consumption as in Diamond and Dybvig (1983). The structure of private

information and the locational itineraries of agents in the model are set up so as to capture the kinds of constraints agents typically face in real-world trading situations. In particular, physical capital in the model takes the form of "fruit trees," which may be either good or bad, and agents have private information about the quality of trees. In decentralized market arrangements, there is an information asymmetry in the trading of claims on fruit trees. Young agents are uncertain about their future demands for liquidity, and claims on trees may be illiquid because of the informational friction.

We first follow an approach similar to that of Townsend (1987, 1988) by invoking the revelation principle (see Harris and Townsend, 1981) in defining a Pareto optimum, with allocations for the private information environment satisfying resource constraints, incentive constraints, and participation constraints. Then, a particular decentralized market arrangement, a *laissez-faire* banking regime, is considered, and it is asked whether such an arrangement supports a Pareto optimal allocation.

In a *laissez-faire* banking regime, trees serve as backing for good and bad circulating notes and for noncirculating notes (bank deposits). Because of private information in the market for circulating notes, good and bad circulating notes sell at the same price. There is then an incentive to plant bad trees and to issue bad circulating notes, in spite of the fact that bad trees yield an inferior long-term rate of return, since bad trees are less costly to plant. Thus, bad circulating notes are much like the "lemons" in Akerlof (1970). In the *laissez-faire* banking regime, there exists no stationary equilibrium with valued fiat money except in a measure-zero subset of the parameter space. If the stationary equilibrium exists (which it may not), it will be one of two types, depending on parameter values. In the first type, a version of Gresham's law holds; the only notes that circulate are those backed by bad capital. Here, there is no misrepresentation of the quality of circulating notes. That is, there is no fraudulent note issue, in spite of the existence of a potential for fraud. The other type of equilibrium involves fraud; good circulating notes coexist with bad ones. In general, the fraction of circulating notes traded which are good increases as bad trees become more costly to plant and as the need for liquidity falls. Both of these effects tend to reduce the incentive to issue bad circulating notes. In equilibrium, bad circulating notes have the highest transactions velocity, followed by good circulating notes (if they are held), followed by bank deposits. Assets are ranked in the reverse order in terms of two-period rates of return (redemption values).

Most (if not all) fiat money regimes place restrictions on the kinds of liabilities private agents may issue. This model shows why these restrictions might be necessary to induce agents to hold fiat money and why they may also be welfare-improving. The *laissez-faire* banking equilibrium, if it exists, is not Pareto optimal, but if there is a legal restriction that bans the

issue of private circulating notes, then there exists a unique stationary equilibrium with valued fiat money that Pareto dominates the laissez-faire banking equilibrium. This stationary monetary equilibrium with legal restrictions is also Pareto optimal within a particular class of allocations. These results hold in spite of the fact that there exist assets in the model that dominate fiat money in rate of return. In the equilibrium with legal restrictions, as in the laissez-faire banking equilibrium, a circulating medium of exchange (fiat money) coexists with bank deposits, and transactions velocity is inversely related (across assets) to rate of return.¹

The remainder of the paper is organized as follows. To ease the reader into the central model of the paper, a simpler version is presented in Section 2, and this model is used to demonstrate some of the important results in an informal manner. However, the simple model of Section 2 cannot address all aspects of the problem at hand. After Section 2, we proceed more formally. A richer, and more novel, model is constructed in Section 3, and in Section 4 a Pareto optimal allocation is defined for this environment. Section 5 examines the laissez-faire banking regime. In Section 6, it is shown that this laissez-faire regime yields a suboptimal equilibrium allocation of resources. Section 7 addresses how legal restrictions permit the existence of a Pareto optimal monetary equilibrium which Pareto dominates the laissez-faire banking equilibrium. The final section summarizes and concludes.

2. A MODEL WITH TWO-PERIOD-LIVED CONSUMERS

Some of the important implications of the model constructed in Section 3 can be more easily understood in a simpler model. Analysis of the simple case also shows why the elements of the richer structure in Section 3 are needed to obtain all results of interest.

We take as our base case a model similar to the overlapping generations model studied by Wallace (1980). Periods are indexed by $t = 1, 2$,

¹ There is some related literature on private information in asset markets and the welfare-improving role of legal restrictions. The model of Aiyagari (1989) illustrates how "lemons" can dominate trade in asset markets with private information. In Smith (1986), it is shown, in an adverse selection environment, that the introduction of fiat money can permit the existence of equilibrium. Azariadis and Smith (1991) also show how adverse selection in credit markets modifies the characteristics of equilibrium in overlapping generations models with pure exchange. In the seignorage model constructed in Bryant and Wallace (1984), legal restrictions can be optimal because they allow the government to price discriminate. Some results similar to those in this paper are in Bernhardt and Engineer (1991). They construct a turnpike model of money, where an equilibrium with valued fiat money may not exist under circumstances where legal restrictions on barter trade can give money value and increase welfare. Williamson and Wright (1991) consider a model where a welfare-improving role for money arises when barter involves qualitative uncertainty.

3, . . . , and there are N_1 agents born in each period, with all agents living for two periods. Each agent has a utility function $V(c_1, c_2)$, where c_i is consumption in the i th period of life, $i = 1, 2$. Assume that $V(\cdot, \cdot)$ is twice continuously differentiable, increasing in both arguments, and strictly concave, and that $\lim_{c_1/c_2 \rightarrow \infty} D_1V/D_2V = 0$ and $\lim_{c_1/c_2 \rightarrow 0} D_1V/D_2V = \infty$. Each agent born in periods $t = 1, 2, 3, \dots$, is endowed with y units of a consumption good when young and nothing when old. There are N_1 one-period-lived agents alive in period 1 who are each endowed with H units of fiat money. Fiat money consists of perfectly divisible nonreproducible pieces of paper.

Each period N_2 one-period-lived agents are born whose utility is increasing in consumption. We will call these one-period-lived agents "banks." Banks have access to two one-period storage technologies. A unit of storage is like a fruit tree, in that some resources are required in the current period to plant the tree, and it yields consumption (fruit) in the following period, after which the tree dies (disintegrates). There are two kinds of trees, good and bad. The cost of planting a good (bad) tree is 1 (β) units of the consumption good, and its return is α_1 (α_2). Assume that $\alpha_2 < \beta < 1 < \alpha_1$. Thus, bad trees are less costly to plant, but they yield a lower one-period rate of return, that is $\alpha_2/\beta < 1 < \alpha_1$. Consumers do not have access to storage technologies.

Each period, banks issue claims to trees, which sell at prices q_t^g (a claim to a good tree issued in period t) and q_t^b (a claim to a bad tree). The bank uses the proceeds from these liability issues to finance the planting of trees and consumption. Given free entry into banking, zero profits in equilibrium imply that $q_t^g = 1$, and $q_t^b = \beta$. Therefore, the rate of return to a consumer from holding a good (bad) bank liability is α_1 (α_2/β). Since $\alpha_2/\beta < \alpha_1$, bad bank liabilities will not be issued in equilibrium, and bad trees will not be planted.

Since the bad storage technology will never be used, we can then show, from Wallace (1980), that a monetary equilibrium does not exist for this environment. That is, if we let p_t denote the price of fiat money in terms of the consumption good, then the only equilibrium is one where $p_t = 0$ for all t . For example, if a stationary monetary equilibrium (an equilibrium where agents in generations $t = 1, 2, 3, \dots$, receive the same consumption allocation) existed, a young agent in period t would face a gross rate of return of $p_{t+1}/p_t = 1$. But $\alpha_1 > 1$, so fiat money is dominated in rate of return by bank liabilities, and this is therefore not an equilibrium. In the unique equilibrium, which is Pareto optimal, young consumers save by holding good bank liabilities, banks plant good fruit trees, and all consumers face a one-period equilibrium gross rate of return of α_1 .

In this environment, laissez-faire banking is Pareto optimal, and introducing legal restrictions which would give fiat money value in equilibrium would yield a suboptimal equilibrium. For example, suppose the govern-

ment prohibits all private money; that is, it bans banking altogether. Then there exist equilibria with valued fiat money; but from Wallace (1980) we know that none of these equilibria are Pareto optimal. One monetary equilibrium is a stationary monetary equilibrium with $p_{t+1}/p_t = 1$, in which some agents could be made better off with no agents becoming worse off, if use were made of the good tree-planting technology.

Now, suppose that we introduce some private information about storage in this environment. That is, assume that only the bank knows the quality of the trees that it plants, so that consumers cannot tell the difference between good and bad bank liabilities. There is no mechanism for punishing banks which plant bad trees, as banks are only one-period-lived, so that a lemons problem exists, as in Akerlof (1970). We assume that, although consumers cannot distinguish among good and bad trees, they know the quality characteristics of the aggregate stock of fruit trees planted.

We confine attention to stationary competitive equilibria, where prices are constant for all t . First, consider stationary nonmonetary equilibria where $p_t = 0$ for all t . In this equilibrium, bank liabilities will be issued, since consumers wish to hold any asset which yields a positive rate of return. Since good and bad bank liabilities are indistinguishable, they sell for the same price, that is $q_t^g = q_t^b = q_t$. Given that bad trees are less costly to plant, a bank receives the largest profit from issuing bad liabilities, so the only bank liabilities issued and trees planted will be bad ones. Zero profits in banking then imply that $q_t = \beta$. The equilibrium one-period return on a bank liability is therefore $\alpha_2/\beta < 1$. In this laissez-faire banking equilibrium, a version of Gresham's law holds; bad private money drives out good private money.

Next, consider stationary monetary equilibria where p_t is constant and positive for all t . The gross rate of return on fiat money is $p_{t+1}/p_t = 1$. Therefore, there does not exist a stationary monetary equilibrium where bank liabilities are held as, in such an equilibrium, fiat money would dominate bank liabilities in rate of return, i.e., $1 > \alpha_2/\beta$, a contradiction. In a stationary monetary equilibrium where no bank liabilities are issued, consumers could identify the quality of the marginal bank liability, as the aggregate quality of trees planted is public knowledge. Thus, in such an equilibrium we can have $q_t^g \neq q_t^b$. Here, there must be nonpositive profits in equilibrium from the issue of either kind of bank liability, that is $q_t^g \leq 1$ and $q_t^b \leq \beta$. Also, if either liability were issued, it could not dominate fiat money in rate of return, that is $\alpha_1/q_t^g \leq 1$, and $\alpha_2/q_t^b \leq 1$. Therefore, $q_t^b \in [\alpha_2, \beta]$ but, since $\alpha_1 > 1$, there does not exist a q_t^g satisfying $q_t^g \leq 1$ and $q_t^g \geq \alpha_1$. Therefore, this equilibrium does not exist, and the unique stationary equilibrium is the Gresham's law equilibrium where the only stores of value are bad bank liabilities. Bad private money not only drives out good private money, but it drives out fiat money as well.

Suppose now that the government prohibits the trading of private bank liabilities. This eliminates the Gresham's law equilibrium, and there now exist two stationary equilibria: an autarkic equilibrium where $p_t = 0$ and all agents consume their endowment and the stationary monetary equilibrium where all consumers save by holding fiat money and face a gross rate of return of 1. The stationary monetary equilibrium Pareto dominates the Gresham's law equilibrium which exists in the absence of legal restrictions. That is, consumers in generations $t = 1, 2, 3, \dots$, face a higher rate of return and one-period-lived agents at $t = 1$ consume more in the former equilibrium. Bank profits are zero in both equilibria.

The results in the private information model conform to what is known about real-world fiat money regimes, in that some intervention is necessary to give fiat money value. To my knowledge, recorded monetary history does not contain an instance where fiat money was valued in the absence of legal restrictions. These restrictions have included the prohibition of private note issue, reserve requirements, and legal tender laws. The fact that prohibiting private "money" can bring about a welfare improvement in the model, by effectively giving the government a monopoly on money production (if we suppose the government has sole access to the technology for producing fiat money), is consistent with Friedman's (1960) arguments. The failure of laissez-faire banking occurs in the private information model for the same reasons as given by Friedman. That is, private information creates an incentive for fraud to occur in the production of private money. However, note that fraud does not occur here in equilibrium. With laissez faire, the Gresham's law equilibrium has the property that all bank liabilities are known to be bad, and no bank could credibly claim otherwise. There can be two kinds of banks, those with good asset portfolios and those with bad ones. Consumers are unable to tell the difference between a good bank and a bad bank, or the cost to doing so is so high that no one is willing to incur it. Since it is more profitable to be a bad bank and because there is no mechanism to enforce good behavior by a bank, all banks will be bad in a laissez-faire equilibrium; i.e., they produce bad money.

Although the model outlined above illustrates some of the main points of this paper, it leaves out some details which are important in studying monetary exchange and legal restrictions on private intermediation. The primary problem with this model is that the only asset characteristic that matters is one-period rate of return. The model thus cannot differentiate between the roles played by circulating private claims and noncirculating claims, or more specifically, between bank notes and bank deposits. As the model does not differentiate between circulating and noncirculating claims, *all* financial intermediary liabilities are subject to a lemons problem, which yields the unpalatable implication that closing down all inter-

mediation can be beneficial. Friedman's arguments for a government monopoly in the production of money hinged on the peculiar private information and enforcement problems associated with circulating claims. Few would argue that these problems would be as significant in the case of a noncirculating asset such as a bank deposit, where the holder of the claim has a greater incentive to acquire knowledge of the quality of assets in the portfolio backing the claim.

In addition to the above problems, the model cannot display rate-of-return dominance. This is a feature which is closely related to the fact that the model cannot distinguish circulating claims from noncirculating ones. That is, any rate-of-return differences among assets must be reflected in differences in these assets along other dimensions. Since there is no important difference among assets, it is impossible to obtain rate-of-return dominance.

3. A MODEL WITH UNCERTAIN-LIVED AGENTS

In this section, the private information environment of Section 2 is enriched in such a way that we can model circulating and noncirculating claims and so that the rate-of-return dominance of circulating media of exchange can be an outcome. As there is more novelty in this environment than in that of the previous section, we proceed more formally.

In each period $t = 1, 2, 3, \dots$, a continuum of agents is born, where the measure of each generation is normalized to unity. Each agent is endowed with y units of a consumption good when young. In middle age, agents learn their type, i , where $i = 1, 2$. A type 1 agent lives for two periods with utility given by $u(c_2)$, and type 2 agents live three periods with utility $v(c_3)$. Here, c_i denotes consumption in the i th period of life. The functions $u(\cdot)$ and $v(\cdot)$ are twice continuously differentiable, increasing, and strictly concave. It is assumed that $u'(0) = v'(0) = \infty$. Nothing of importance changes if agents consume in the first period of life. The parameter π denotes the fraction of agents in a generation who are type 1 and also denotes the probability of being type 1 for any agent. We have $0 < \pi < 1$.

Locations where production occurs are indexed by $t = 1, 2, 3, \dots$, and there is another location, denoted location 0, the role of which is made clear shortly. Agents move among locations according to the following itineraries. At the start of period t , each young member of generation t is confined to location t , where she can plant fruit trees using her endowment. There are good fruit trees and bad fruit trees, with tree-planting technologies and fruit yields identical in most respects to those in the model of the previous section. The only difference here is that a tree

planted in period t yields fruit in period $t + 2$ at location t . That is, fruit has a longer gestation period than in the previous model, and a tree is confined to a particular location. At the end of each period, all agents currently alive meet at location 0. Consumption goods (endowments and fruit) can be consumed at any time during the period, but they perish at the end of the period. At the beginning of each period, middle-aged agents learn their type, i , and middle-aged and old-aged agents go to location $t - 2$, which is where trees yield fruit in the current period. Thus, young agents are separated from old and middle-aged agents at the beginning of each period.

The environment has the following information structure. Before a tree yields fruit, its type is observable only to the agent who plants it, but fruit yields are public information. Agents can observe trees at all locations without observing their types, and the agents present at location t at the beginning of period t can observe trees planted and who plants them. At the beginning of period $t + 1$, the fraction of trees of each type planted in period t becomes public information, although the type of each tree remains the private information of the agent who planted it. At location t , there exists a record-keeping technology which tracks which agents planted which trees. If a generation t agent chooses to submit to the technology in period $t + 2$, she can be identified, and therefore associated with the trees she planted in period t . There is an identification technology at location t which can verify the ages of agents willing to submit to it. Consumption by agents at location t is unobservable. At location 0, type and age are private information.

The information/spatial structure allows agents to commit to arrangements which might allow for the planting of good trees, in contrast to the private information model of the previous section. That is, if an agent born in period t is a type 2, and returns to location t in period $t + 2$ and submits to the record-keeping technology, the type of trees she planted in period t can be verified. However, if an agent is type 1, to consume she will have to trade at location 0, where other agents know little about her and the objects she can offer in exchange. The interactions at production locations are meant to capture the nature of relationships between banks and their customers, while the interaction among agents at location 0 is subject to the kinds of information constraints that agents typically face in carrying out transactions on decentralized markets.

In period 1, there are π middle-aged type 1 agents, $1 - \pi$ middle-aged type 2 agents, and $1 - \pi$ old agents, each of whom is endowed with H units of fiat money. The initial old and middle-aged follow the same itineraries as agents in other generations. Agents who are type 1 and middle-aged and those who are old-aged in period 1 maximize period 1 consumption, while type 2 middle-aged agents maximize period 2 consumption.

4. PARETO OPTIMAL ALLOCATIONS

In defining a Pareto optimal allocation, we invoke the *revelation principle* (see Harris and Townsend, 1981), restricting attention to allocations where it is not in any agent's interest to misrepresent age, type, or production. Let θ_t denote an agent's vector of characteristics in period t , where $\theta_t = (s, j)$ with $s \in \{-1, 0, 1, \dots\}$ and $j \in \{0, 1, 2\}$. Here, s denotes an agent's generation, and j denotes type, where $j = 0$ for a young agent whose type has not yet been revealed. Also, note that $s = -1$ for agents who are old at $t = 1$, and $s = 0$ for agents who are middle-aged at $t = 1$. An allocation is proposed at $t = 1$ for $t = 1, 2, 3, \dots$. Young agents who choose to participate in period t each plant k_t^g good fruit trees and k_t^b bad fruit trees, absorbing $k_t^g + \beta k_t^b$ units of their endowment. What remains of the endowments of young agents is then distributed among the agents at location 0 at the end of period t , with an agent with characteristics θ_t receiving $x_t(\theta_t)$ units of the consumption good. At location $t - 2$ at the beginning of period t , where $t = 3, 4, 5, \dots$, each agent present receives $z_t(\theta_t)$ units of fruit. The planner then faces the following resource constraints. First, resources absorbed in planting trees plus consumption of agents at location 0 cannot exceed endowments. That is,

$$k_t^g + \beta k_t^b + \pi x_t(t - 1, 1) + (1 - \pi)x_t(t - 2, 2) \leq y, \quad (1)$$

$t = 1, 2, 3, \dots$. Second, consumption at location $t - 2$ cannot exceed the production of fruit. That is,

$$\pi z_t(t - 1, 1) + (1 - \pi)z_t(t - 2, 2) \leq \alpha_1 k_{t-2}^g + \alpha_2 k_{t-2}^b, \quad (2)$$

$t = 3, 4, 5, \dots$.

The expected utility of a generation t agent from the proposed allocation is given by

$$U_t = \pi u[x_{t+1}(t, 1) + z_{t+1}(t, 1)] + (1 - \pi)v[x_{t+2}(t, 2) + z_{t+2}(t, 2)]. \quad (3)$$

A proposed allocation must satisfy certain incentive constraints. First, it must be in a young agent's interest to plant fruit trees as directed by the planner. It can be observed whether the correct number of trees is planted by any agent, but a tree's type cannot be verified until it bears fruit two periods later. A young agent could cheat by planting bad trees if directed to plant good trees but, since young agents get no utility from consumption, there is no gain from doing so. Therefore, young agents will plant trees as directed by the planner. Second, it must be in the interest of each agent at location 0 at the end of period t to report her true characteristics.

Let $\tilde{\theta}_t \neq \theta_t$. Then, the incentive constraint is

$$x_t(\theta_t) \geq x_t(\tilde{\theta}_t), \quad \theta_t, \tilde{\theta}_t = (t-1, 1), (t-2, 2); \quad (4)$$

that is, an agent cannot receive a larger quantity of consumption goods at location 0 by misreporting her type.

Agents can choose when young whether or not to participate in the proposed allocation mechanism, by not planting trees as directed by the planner and not submitting their endowments to the planner when young. If she does not participate, there is still the possibility that an agent can consume in middle or old age by posing at location 0 as a participant. However, nonparticipation by any agent can be detected by the planner, since it is known how many young agents are participating in directed tree planting. Should any member of generation t not participate, the planner imposes the harshest penalty possible on all members of generation t , by setting $x_{t+1}(t, 1) = x_{t+2}(t, 2) = z_{t+1}(t, 1) = z_{t+2}(t, 2) = 0$. The allocation must then satisfy the participation constraint

$$U_t \geq \pi u(0) + (1 - \pi)v(\alpha_1 y), \quad (5)$$

where the right side of the inequality is the expected utility the agent would obtain in autarky, by investing optimally in tree planting. That is, if the agent does not participate, it is optimal to plant y good trees and to consume the resulting fruit production. Given the penalty imposed by the planner, the agent consumes zero if she is type 1.

DEFINITION 1. A Pareto optimal allocation $\{x_t(\theta_t), z_t(\theta_t), k_t^g, k_t^b, U_t\}$ satisfies (1)–(5) and has the property that there is no alternative allocation $\{\bar{x}_t(\theta_t), \bar{z}_t(\theta_t), \bar{k}_t^g, \bar{k}_t^b, \bar{U}_t\}$ satisfying (1)–(5) and

$$\bar{U}_t \geq U_t, \quad t = 1, 2, 3, \dots, \quad (6)$$

$$\bar{x}_1(-1, 2) \geq x_1(-1, 2), \quad (7)$$

$$\bar{x}_1(0, 1) \geq x_1(0, 1), \quad (8)$$

$$\bar{x}_2(0, 2) \geq x_2(0, 2), \quad (9)$$

and at least one of (6)–(9) with strict inequality (for any t in the case of (6)). (10)

Except for the incentive and participation constraints, this definition of Pareto optimality is standard (see Wallace, 1980).

5. LAISSEZ-FAIRE BANKING EQUILIBRIUM

The objective of this section is to determine whether a Pareto optimal allocation can be supported as a competitive equilibrium with uncon-

strained private banking. If attention is confined to equilibria that are stationary (in a sense to be defined), then the answer is no.

In addition to fiat money, let there be two kinds of securities, circulating notes and noncirculating notes. A circulating note is a piece of paper which entitles its bearer to the fruit from a particular tree. These notes are issued by the agent who plants the tree and are certifiably backed by this tree, perfectly divisible, and not counterfeitable. If a circulating note issued in period t is good, i.e., backed by a good tree, then it is redeemable for α_1 units of fruit in period $t + 2$ at location t , whereas a bad circulating note is redeemable for α_2 units. Given the environment, good and bad circulating notes are distinguishable only to the agent who issues them. Noncirculating notes are intermediated by a bank. A noncirculating note issued in period t is a piece of paper exchanged by the bank for a claim to the fruit from a tree at location t in period $t + 2$. Noncirculating notes issued in period t can be redeemed in period $t + 2$ only by the agents to whom they were originally issued, and like circulating notes they cannot be counterfeited. The redemption value of a noncirculating note is zero if the yield from the agent's fruit tree indicates the tree was bad. However, if an agent exchanges a claim to a good tree for a noncirculating note in period t and then returns the note for redemption in period $t + 2$, the agent receives a proportional share of the return on the bank's portfolio. Given the above punishment mechanism, agents will exchange only claims to good trees for noncirculating notes. If the bank issues notes to many agents, by the law of large numbers the fraction of agents who will redeem noncirculating notes is the fraction of type 2 agents in the population, $1 - \pi$. Therefore, with zero profits in banking, the redemption value of a noncirculating note is $\alpha_1/(1 - \pi)$.

Circulating notes in this model share some of the features associated with the bank notes issued during laissez-faire regimes such as the U.S. free banking period. In particular, they can be issued in small denominations, and redemption value does not depend on who holds the note. Unlike the securities issued by U.S. free banks, though, these notes are not redeemable at any time at their par value, but look more like shares in a mutual fund. However, this is not out of line with the views of Rolnick and Weber (1987), who argue that free bank notes were priced to reflect their perceived backing and that free banks thus functioned much like mutual funds. With regard to the fixed redemption period of the notes in this model, it is useful from an analytical point of view to abstract from agents' redemption decisions. Also, this shows that problems can arise with laissez-faire banking that have nothing to do with "overissue" of circulating notes or stochastic redemption.

Noncirculating notes have several features associated with bank deposits subject to withdrawal or to payment by check. First, there are contingencies under which these assets cannot be used to carry out trans-

actions. If an agent learns she is type 1 in middle age, then a noncirculating note is worthless but a circulating note or fiat money will not be, provided that some agent is willing to accept these other assets in exchange. Second, noncirculating notes do not change hands just as, under normal circumstances, checks are redeemed immediately and do not circulate. Third, there is a need here, as in the case of deposit banking, for the bank to diversify across depositors to assure a predictable pattern of withdrawals (i.e., redemptions).²

There appear to be no other banking-type securities which we have omitted from the analysis. For example, one might suppose that, since agents have uncertain preferences as in Diamond and Dybvig (1983), there would be room here for Diamond–Dybvig-type deposit contracts which would allow agents to insure against preference shocks without holding circulating notes. However, such contracts would require that middle-aged agents get together with young agents at a location where the middle-aged agents could be identified; since this is not possible here, such contracts are infeasible.

Let p_t denote the price of fiat money in terms of the consumption good, q_t the acquisition price of a secondhand circulating note (i.e., a circulating note issued in period $t - 1$), q_t^g (q_t^b) the price a seller gets for a good (bad) secondhand circulating note, and r_t the price of a newly issued circulating note. An agent born in period t chooses nominal fiat money balances m_t^t , m_{t+1}^t , noncirculating notes d_t , new issues of good circulating notes g_t^t , new issues of bad circulating notes b_t^t , purchases of newly issued circulating notes w_{t+1}^t , sales of good circulating notes a_t^t , a_{t+1}^t , sales of bad circulating notes f_t^t , f_{t+1}^t , and acquisitions of secondhand notes s_t^t , s_{t+1}^t to maximize expected utility. A subscript denotes the date at which the asset is held or the date when an acquisition is made, and a superscript denotes the agent's date of birth. At the time of issue, a circulating note can be held by the issuer or sold, and it may be sold in middle age. A secondhand circulating note is a note issued by another agent. The fraction of traded secondhand circulating notes that are good is denoted by γ_t , where t is the date when these notes were newly issued. Agents who buy secondhand circulating notes diversify across these notes so as to obtain the mean redemption value, which is $\gamma_t\alpha_1 + (1 - \gamma_t)\alpha_2$. Let c_t^s denote the consumption of a generation s agent in period t . An agent born in period t then solves

$$\max[\pi u(c_{t+1}^t) + (1 - \pi)v(c_{t+2}^t)] \quad (11)$$

² If the reader remains unconvinced that the noncirculating notes in the model correspond to the deposit liabilities of banks, note that this correspondence is not critical to the main points of the paper. What is of primary importance is that noncirculating notes are alternative assets which are imperfect substitutes for circulating media of exchange.

subject to:

$$0 \leq y - p_t m_t^i - g_t^i - \beta b_t^i + r_t(a_t^i + f_t^i) - d_t - q_t s_t^i, \quad (12)$$

$$c_{t+1}^i \leq p_{t+1} m_t^i + q_{t+1}^g (g_t^i - a_t^i) + q_{t+1}^b (b_t^i - f_t^i) + [\gamma_{t-1} \alpha_1 + (1 - \gamma_{t-1}) \alpha_2] s_t^i, \quad (13)$$

$$p_{t+1} m_{t+1}^i + q_{t+1} s_{t+1}^i + r_{t+1} w_{t+1}^i \leq p_{t+1} m_t^i + q_{t+1}^g a_{t+1}^i + q_{t+1}^b f_{t+1}^i + [\gamma_{t-1} \alpha_1 + (1 - \gamma_{t-1}) \alpha_2] s_t^i, \quad (14)$$

$$c_{t+2}^i \leq p_{t+2} m_{t+1}^i + q_{t+2} w_{t+1}^i + [\alpha_1 / (1 - \pi)] d_t + \alpha_1 (g_t^i - a_t^i - a_{t+1}^i) + \alpha_2 (b_t^i - f_t^i - f_{t+1}^i) + [\gamma_t \alpha_1 + (1 - \gamma_t) \alpha_2] s_{t+1}^i, \quad (15)$$

$$a_t^i \leq g_t^i, \quad (16)$$

$$f_t^i \leq b_t^i, \quad (17)$$

$$g_t^i + \beta b_t^i + d_t \leq y, \quad (18)$$

$$a_{t+1}^i \leq g_t^i - a_t^i, \quad (19)$$

$$f_{t+1}^i \leq b_t^i - f_t^i, \quad (20)$$

$$c_t^i, c_{t+1}^i, c_{t+2}^i, m_t^i, m_{t+1}^i, d_t, g_t^i, b_t^i, w_{t+1}^i, a_t^i, a_{t+1}^i, f_t^i, f_{t+1}^i, s_t^i, s_{t+1}^i \geq 0. \quad (21)$$

Here, (12)–(15) are the budget constraints faced by a young agent, a middle-aged type 1 agent, a middle-aged type 2 agent, and an old agent, respectively. Inequalities (16) and (17) state that sales of good and bad circulating notes when young cannot exceed circulating note issues, while (18) states that circulating and noncirculating note issues when young cannot exceed the agent's endowment. In (19) and (20), sales of circulating notes in middle age cannot exceed what remains from the previous period given circulating note issues and sales when young. Finally, (21) specifies nonnegativity constraints.

DEFINITION 2. A *laissez-faire banking equilibrium* (LE) is a nonnegative sequence of prices $\{\bar{p}_t, \bar{q}_t, \bar{q}_t^g, \bar{q}_t^b, \bar{r}_t\}$, a nonnegative sequence of asset holdings, asset purchases, and asset sales $\{\bar{h}_t\} = \{\bar{m}_t^i, \bar{m}_{t+1}^i, \bar{d}_t, \bar{g}_t^i, \bar{b}_t^i, \bar{w}_{t+1}^i, \bar{a}_t^i, \bar{a}_{t+1}^i, \bar{f}_t^i, \bar{f}_{t+1}^i, \bar{s}_t^i, \bar{s}_{t+1}^i\}$, a nonnegative sequence of consumption quantities $\{\bar{c}_t\} = \{\bar{c}_{t+1}^i, \bar{c}_{t+2}^i\}$, and a sequence of γ 's, $\{\bar{\gamma}_t\}$, such that

(i) $\bar{c}_t$ and $\bar{h}_t$ are chosen to solve (11) subject to (12)–(21), given $\{p_t, q_t, q_t^g, q_t^b, r_t\} = \{\bar{p}_t, \bar{q}_t, \bar{q}_t^g, \bar{q}_t^b, \bar{r}_t\}$.

(ii) The money market clears.

$$\bar{p}_t [\bar{m}_t^i + (1 - \pi) \bar{m}_{t-1}^i] = \bar{p}_t (2 - \pi) H, \quad t = 2, 3, 4, \dots \quad (22)$$

$$\bar{p}_1 m_1^i = \bar{p}_1 H. \quad (23)$$

(iii) The market for new circulating notes clears.

$$(1 - \pi)\bar{w}_t^{t-1} = \bar{a}_t^t + \bar{f}_t^t, \quad t = 1, 2, 3, \dots \quad (24)$$

(iv) The market for secondhand circulating notes clears.

$$\bar{s}_t^t + (1 - \pi)\bar{s}_t^{t-1} = \pi(\bar{g}_{t-1}^{t-1} - \bar{a}_{t-1}^{t-1} + \bar{b}_{t-1}^{t-1} - \bar{f}_{t-1}^{t-1}) \\ + (1 - \pi)(\bar{a}_{t-1}^{t-1} + \bar{f}_{t-1}^{t-1}) + (1 - \pi)w_{t-1}^{t-2}, \quad t = 3, 4, 5, \dots \quad (25)$$

$$\bar{s}_2^2 + (1 - \pi)\bar{s}_2^1 = \pi(\bar{g}_1^1 - \bar{a}_1^1 + \bar{b}_1^1 - \bar{f}_1^1) + (1 - \pi)(\bar{a}_2^1 + \bar{f}_2^1). \quad (26)$$

(v) The sequence of γ 's is correctly anticipated. That is, if either $g_t^t > 0$ or $b_t^t > 0$, then

$$\gamma_t = [\pi g_t^t + (1 - \pi)a_t^t + (1 - \pi)a_{t+1}^t] / [\pi b_t^t + (1 - \pi)f_t^t + (1 - \pi)f_{t+1}^t \\ + \pi g_t^t + (1 - \pi)a_t^t + (1 - \pi)a_{t+1}^t], \quad t = 2, 3, 4, \dots, \quad (27)$$

and, if either $g_1^1 > 0$ or $b_1^1 > 0$, then

$$\gamma_1 = [\pi(g_1^1 - a_1^1) + (1 - \pi)a_2^1] / [\pi(b_1^1 - f_1^1) \\ + (1 - \pi)f_2^1 + \pi(g_1^1 - a_1^1) + (1 - \pi)a_2^1]. \quad (28)$$

If $g_t^t = b_t^t = 0$, then

$$\gamma_t = \text{argmax}[\gamma_t \alpha_1 / q_{t+1}^g + (1 - \gamma_t) \alpha_2 / q_{t+1}^b], \quad t = 1, 2, 3, \dots \quad (29)$$

(vi) Good and bad secondhand notes are indistinguishable in period $t + 1$ if any circulating notes were issued in period t . Otherwise, secondhand notes are distinguishable at the margin. That is, if $g_t^t > 0$ or $b_t^t > 0$, then

$$q_{t+1}^g = q_{t+1}^b = q_{t+1}, \quad (30)$$

and, if $g_t^t = b_t^t = 0$, then

$$q_{t+1} = \text{argmax}_{q_{t+1}^g, q_{t+1}^b} (\alpha_1 / q_{t+1}^g, \alpha_2 / q_{t+1}^b). \quad (31)$$

For the most part, Definition 2 is a standard definition of rational expectations equilibrium. However, there are some important differences here. First, agents' expectations of quantities traded matter. Part (v) of the definition specifies that agents correctly anticipate the quantities of good and bad circulating notes traded in equilibrium. If no circulating notes are issued in period t in equilibrium, then at the margin a secondhand note could be identified as being good or bad, since agents know in period $t + 1$ how many good and bad trees were planted in period t . Given the choice, agents would then prefer to hold whichever type of secondhand circulating note gave the highest one-period rate of return, which implies (29).

Second, in part (vi) of the definition, we need to distinguish between situations where, on the one hand, prices of good and bad secondhand circulating notes can differ and, on the other hand, these prices cannot differ. If any circulating notes were issued in period t , then in period $t + 1$, the prices received by sellers of good and bad circulating notes must be identical, and equal to the acquisition price of one of these notes. However, if no circulating notes were issued in period t then, in period $t + 1$, good and bad circulating notes would be distinguishable at the margin if they had been issued in period t . Therefore, their prices may differ, and the acquisition price, q_{t+1} , can be defined to be the price of the type of secondhand circulating note that gives the highest one-period rate of return.

As a first step in characterizing an equilibrium, we can show, in the case where $b'_t > 0$ or $g'_t > 0$ (circulating notes are issued in period t), that a number of choice variables will be set to zero in the solution to the optimization problem, (11) subject to (12)–(21). First, since newly issued good and bad circulating notes sell at the same price, no young agent would sell a new good circulating note, so that $a'_t = 0$ for all t . Second, if $b'_t > 0$ or $g'_t > 0$, the one-period return for a young agent to holding a newly issued good circulating note is q_{t+1} , and the one-period return to holding a newly issued bad circulating note is $q_{t+1}/\beta > q_{t+1}$ for $q_{t+1} > 0$, so no young agent would issue a good circulating note with the intention of selling it in middle-age if type 2. Thus, $a'_{t+1} = 0$. Third, if $b'_t > 0$ or $g'_t > 0$, then $f'_{t+1} = b'_t - f'_t$; that is, newly issued bad circulating notes held by a young agent at the end of the first period of life are all sold in middle age. To see this, note first that $q_{t+1} > 0$ is necessary for bad circulating notes to be issued in period t , since if $q_{t+1} = 0$ then noncirculating notes dominate bad circulating notes in rate of return. If an agent who is middle-aged and type 2 in period t and holds a bad circulating note issued in period $t - 1$ holds the note until maturity then the two-period return is α_2/β . However, if the agent sells the note on the secondhand market in middle age and uses the proceeds to purchase more secondhand notes which are then held to maturity, the two-period return is $[\gamma_t \alpha_1 + (1 - \gamma_t) \alpha_2]/\beta$. Therefore, if any good secondhand notes are traded, the second strategy strictly dominates the first, and the two strategies are equivalent if only bad secondhand notes are traded ($\gamma_t = 0$).

Optimizing behavior then implies, from the above, (27), and (28), that, if $g'_t > 0$ or $b'_t > 0$, then

$$\gamma_t = \pi g'_t / (\pi g'_t + b'_t). \quad (32)$$

Since all newly issued circulating notes sold are bad, arbitrage gives $r_t = \beta$ for all t . When agents are young, they acquire noncirculating notes, secondhand circulating notes, good and bad newly issued circulating

notes, and fiat money. In middle age, if the agent is type 1, she sells all her circulating notes and fiat money to consume, and her noncirculating notes are worthless. If she is type 2 in middle age, she sells all bad circulating notes issued in the previous period, redeems circulating notes acquired secondhand in the previous period, and reshuffles her portfolio. When old, all notes and fiat money are sold or redeemed, and the agent consumes. Let R_t denote the maximum one-period gross rate of return faced by agents at time t , in an equilibrium where circulating notes are issued in period t . Then, $R_t = \max\{p_{t+1}/p_t, q_{t+1}/\beta, [\gamma_{t-1}\alpha_1 + (1 - \gamma_{t-1})\alpha_2]/q_t\}$, the maximum of gross rates of return on fiat money, newly issued bad circulating notes, and secondhand circulating notes. Then, from (11)–(21), (30), and the Kuhn–Tucker conditions, optimization implies

$$-\lambda + \pi q_{t+1}u' + (1 - \pi)\alpha_1 v' \leq 0, \quad (33)$$

$$-\lambda + \pi R_t u' + (1 - \pi)R_t R_{t+1} v' \leq 0, \quad (34)$$

$$-\lambda + \alpha_1 v' \leq 0. \quad (35)$$

In (33)–(35), λ is the Lagrange multiplier associated with the budget constraint of the young, (12). The left side of (33) is the marginal expected utility from issuing good circulating notes when young, the left side of (34) is the marginal expected utility from acquiring the asset with the highest one-period return when young, and the left side of (35) is the marginal expected utility from acquiring a noncirculating note when young.

The following restriction on preferences and technology is imposed:

$$\alpha_1 v'(\alpha_1 x) \geq bu'(bx), \quad \text{with } x > 0, \quad 0 < b \leq 1. \quad (36)$$

This condition guarantees that the marginal utility of consumption for a type 2 agent is sufficiently large relative to the marginal utility of consumption for a type 1 agent. This inequality rules out some types of equilibria; essentially, it will imply that, in a stationary LE (where it will be made precise below what is meant by a stationary equilibrium), noncirculating notes will always be held. Suppose that $u(x) = x^{1-\mu}/(1 - \mu)$ and $v(x) = \delta x^{1-\mu}/(1 - \mu)$, where μ is the coefficient of relative risk aversion and δ is the discount factor, with $0 < \delta \leq 1$ and $\mu > 0$. Then, condition (36) gives $\delta \geq (b/\alpha_1)^{1-\mu}$. Thus, given α_1 and μ , in this case, condition (36) puts a lower bound on the discount factor, and this lower bound is less than or equal to one if and only if $\mu \leq 1$.

PROPOSITION 1. *In a LE, if good circulating notes are issued in period t , then bad circulating notes are also issued in period t .*

Proof. Suppose that good circulating notes are issued in a LE in period t , but bad circulating notes are not. Then, the two-period gross rate of

return to acquiring newly issued bad circulating notes in period t , selling them in period $t + 1$, and using the proceeds to acquire secondhand notes which are redeemed in period $t + 2$ is $\alpha_1/\beta > \alpha_1$. Also, the one-period rate of return to issuing bad circulating notes at t and selling them at $t + 1$ is $q_{t+1}/\beta > q_{t+1}$. Thus, bad circulating notes dominate good circulating notes in rate of return, a contradiction. Q.E.D.

Given the configuration of securities, Proposition 1 points out an important effect of private information in the model. If all information were public, then good and bad circulating notes would sell at different prices in any equilibrium. Therefore, bad circulating notes could not coexist with good circulating notes in equilibrium, since they yield an inferior return when held from time of issue to maturity.

In what follows, attention is restricted to a particular class of equilibria.

DEFINITION 3. A *stationary laissez-faire banking equilibrium* (SLE) is an equilibrium satisfying Definition 2 with prices and consumption allocations which are time-stationary. That is, $(p_t, q_t, q_t^g, q_t^b) = (p, q, q^g, q^b)$ and $(c_{t+1}^i, c_{t+2}^i) = (c_2, c_3) \forall t$, where p, q, q^g, q^b , and $c_i, i = 2, 3$, are nonnegative constants.

5a. Nonmonetary Equilibrium

We first examine SLEs where fiat money is not valued; that is $p_t = 0$ for all t . We require some notation for characterizing equilibria. Let A denote the set of assets that are held in a SLE, where $A \subset \{G, B, D\}$. If $G \in A$, then good circulating notes are held in equilibrium; if $B \in A$, then bad circulating notes are held; if $D \in A$, then noncirculating notes are held. Given Proposition 1, the only possibilities in a nonmonetary SLE are

- Case N1. $A = \{D\}$
- Case N2. $A = \{B\}$
- Case N3. $A = \{B, D\}$
- Case N4. $A = \{G, B\}$
- Case N5. $A = \{G, B, D\}$.

PROPOSITION 2. Case N1, N2, and N4 SLEs do not exist. A Case N3 SLE exists if and only if

$$\alpha_1(\beta - \pi) - \alpha_2(1 - \pi) \leq 0. \quad (37)$$

A Case N5 SLE exists if and only if

$$\alpha_1(\beta - \pi) - \alpha_2(1 - \pi) \geq 0, \quad (38)$$

and

$$\alpha_1(\beta - \pi) - \beta(1 - \pi) \leq 0. \quad (39)$$

Proof. See the Appendix.

In a Case N1 SLE, the supplies of newly issued and secondhand circulating notes are zero, and fiat money is not valued. Thus, there is no asset in such an equilibrium that allows for strictly positive consumption in middle age. This implies that, for any prices of secondhand circulating notes, q^s and q^b , there is excess demand for some asset, and therefore this cannot be an equilibrium. In the remaining cases, N2, N3, N4, and N5, bad circulating notes are held and there is positive trade on the market for secondhand circulating notes. Therefore, from (30) we have $q_t^s = q_t^b = q_t = q$ for all t , and

$$R_t = q/\beta = [\gamma_t \alpha_1 + (1 - \gamma_t) \alpha_2]/q, \quad (40)$$

for all t . Therefore, $\gamma_t = \gamma$ for all t , and

$$q = \{\beta[\gamma \alpha_1 + (1 - \gamma) \alpha_2]\}^{1/2}. \quad (41)$$

For Cases N2 and N3, we have $\gamma = 0$, so that, from (41), $q = (\beta \alpha_2)^{1/2}$. In a Case N2 equilibrium, given the assumptions on preferences and technology and the price of secondhand circulating notes from (41), agents will have an incentive to hold noncirculating notes, and therefore this cannot be an equilibrium. In a Case N3 SLE, the three assets, good circulating notes, bad circulating notes, and noncirculating notes, have one- and two-period returns per unit invested of $[(\alpha_2 \beta)^{1/2}, \alpha_1]$, $[(\alpha_2/\beta)^{1/2}, (\alpha_2/\beta)]$, and $[0, \alpha_1/(1 - \pi)]$, respectively, where the first element in each ordered pair is the one-period return and the second element is the two-period return. In Fig. 1, V represents the consumption allocation which can be achieved with noncirculating notes only, W the allocation with bad circulating notes, and X the allocation with good circulating notes. Here, VWY is a young agent's budget constraint. Condition (37) guarantees that X lies on or below the line VW . That is, given (37), some convex combination of noncirculating notes and bad circulating notes weakly dominates good circulating notes. Condition (36) guarantees that agents never choose point W (Case N2), but will always choose a point between V and W , such as point Z .

In Cases N4 and N5, both good and bad circulating notes are issued, and $\bar{\alpha} = \gamma \alpha_1 + (1 - \gamma) \alpha_2$, where $\bar{\alpha}$ is the average redemption value of circulating notes. In a Case N4 SLE, (33) and (34) hold with equality.

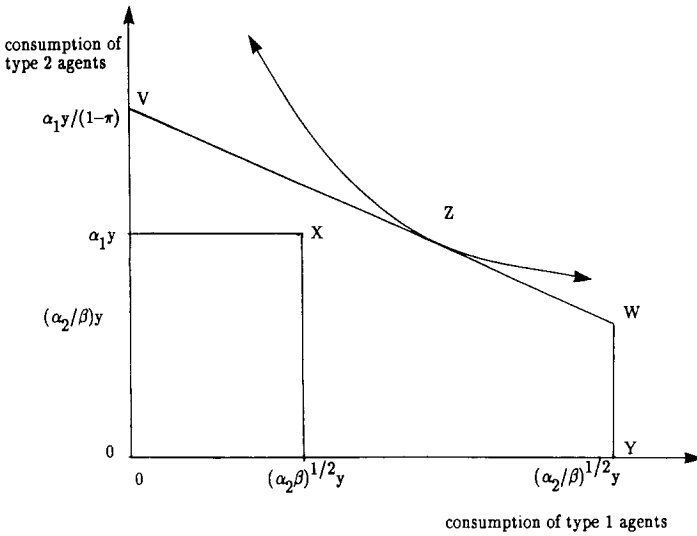


FIG. 1. Gresham's law SLE (Case N3).

Substituting using (33) and (34) in (35), we obtain

$$\alpha_1(\beta - \pi) - \bar{\alpha}(1 - \pi) \geq 0. \quad (42)$$

From (42), in a Case N4 SLE the budget constraint of a young agent looks like $VXWY$ in Fig. 2, where X is on or above the line VW . In Fig. 2, V is the consumption allocation which can be achieved with noncirculating notes alone, X the allocation with good circulating notes, and W the allocation with bad circulating notes. If a Case N4 equilibrium existed, agents facing the budget constraint in Fig. 2 would choose a point in the interior of XW . However, preferences are such that, from (36), agents prefer an allocation on VX , for example point Z in Fig. 2. Therefore a Case N4 equilibrium never exists.

Figure 3 depicts a Case N5 SLE, where good and bad circulating notes coexist with noncirculating notes. Point V denotes the consumption allocation which a young agent can attain by holding only noncirculating notes, X is the allocation with good circulating notes, and W is the allocation with bad circulating notes. A young agent's budget constraint is VWY . In this equilibrium, X must lie on the line VW ; that is we must have

$$\alpha_1(\beta - \pi) - \bar{\alpha}(1 - \pi) = 0, \quad (43)$$

where $\bar{\alpha} = \gamma\alpha_1 + (1 - \gamma)\alpha_2$, and

$$\gamma = [\alpha_1(\beta - \pi) - \alpha_2(1 - \pi)] / (1 - \pi)(\alpha_1 - \alpha_2) \equiv \gamma^*. \quad (44)$$

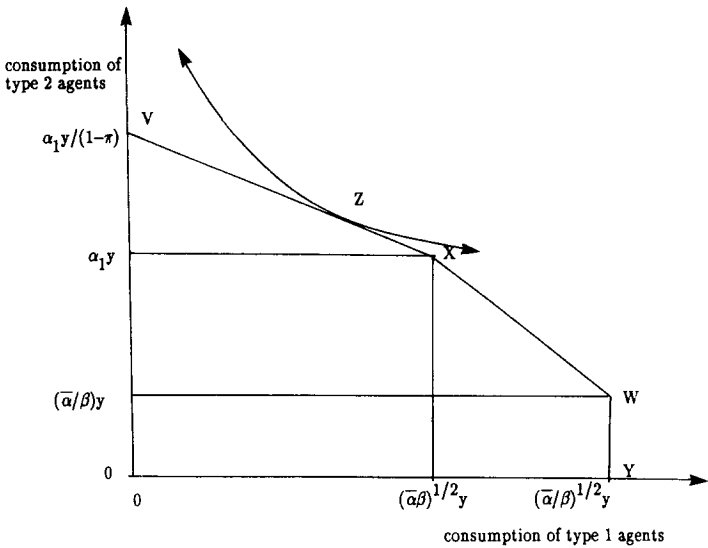


FIG. 2. Case N4 SLE.

For the equilibrium to exist, it must be the case that a young agent's indifference curve is tangent to VW at a point like Z , which is between V and X . Condition (36) guarantees that this will be the case. Conditions (38) and (39) guarantee that $\bar{\alpha} \in [\alpha_2, \alpha_1]$ and that each asset quantity is non-negative along the equilibrium path.

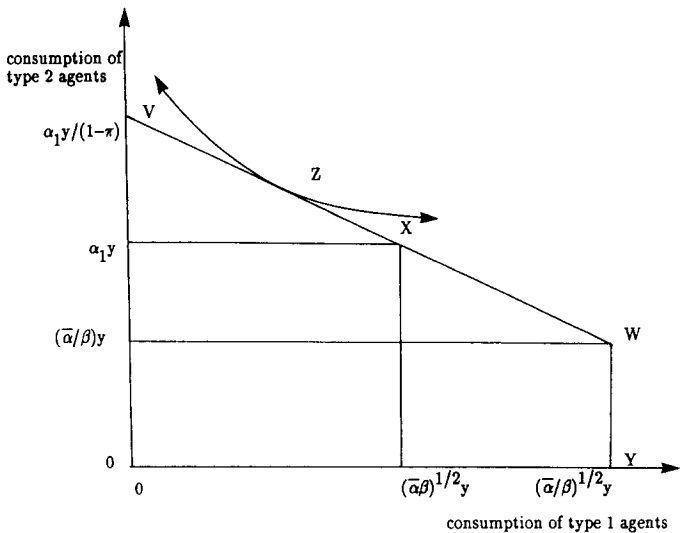


FIG. 3. SLE with fraud (Case N5).

5b. *Monetary Equilibrium*

Still restricting attention to SLEs, we now look for monetary equilibria, where $p > 0$. We therefore have $p_{t+1}/p_t = 1$ for $t = 1, 2, 3, \dots$. We have the following proposition.

PROPOSITION 3. *In a monetary SLE, good and bad circulating notes and noncirculating notes are held. The equilibrium exists if and only if*

$$\alpha_1(\beta - \pi) - \beta(1 - \pi) = 0. \quad (45)$$

*Given (45), there exists a continuum of monetary SLEs, indexed by $p \in (0, \tau^*y/H)$, where $0 < \tau^* < 1$.*

Proof. See the Appendix.

Thus, a monetary SLE does not exist, except in a subset of the parameter space which has zero measure. There cannot be a monetary SLE where bad circulating notes are issued and good circulating notes are not, as bad circulating notes would then be dominated in rate of return by fiat money. There also cannot be a monetary SLE where no circulating notes are issued. In this case, circulating notes could be identified, at the margin, if they were issued. Therefore, the prices of good and bad circulating notes can differ, and there exists no price for good circulating notes, q_t^g , such that no agent wishes to issue such a note, and where the rate of return on the note would not dominate the rate of return on fiat money. This leaves the case where good and bad circulating notes coexist in equilibrium with valued fiat money, and each young agent faces the budget constraint VXW in Fig. 4. Here, V is the consumption allocation which could be attained by holding noncirculating notes only, X the allocation with good circulating notes, and W the allocation with fiat money, bad circulating notes, and secondhand circulating notes. For X to be on the line VW , we must have $\alpha_1(\beta - \pi) - \beta(1 - \pi) = 0$; i.e., this equilibrium exists as a hairline case. If the monetary SLE does exist, the price level is indeterminate because, given the rates of return that agents face, equilibrium is consistent with different quantities of real fiat money balances, with the ratio of good to bad circulating notes constant across equilibria.

In the case where (37) holds as a strict inequality, the SLE features a version of Gresham's law. That is, the only media of exchange are bad circulating notes, which drive out good circulating notes and fiat money. Alternatively, if (38) holds as a strict inequality, and (39) holds, then good and bad circulating notes coexist in the SLE. This second type of equilibrium is one with fraud, in that an agent who circulates a bad note must misrepresent the quality of the assets that are backing the note. Both the Gresham's law equilibrium and the equilibrium with fraud are characterized by a "lemons" problem, as in Akerlof (1970). That is, circulating

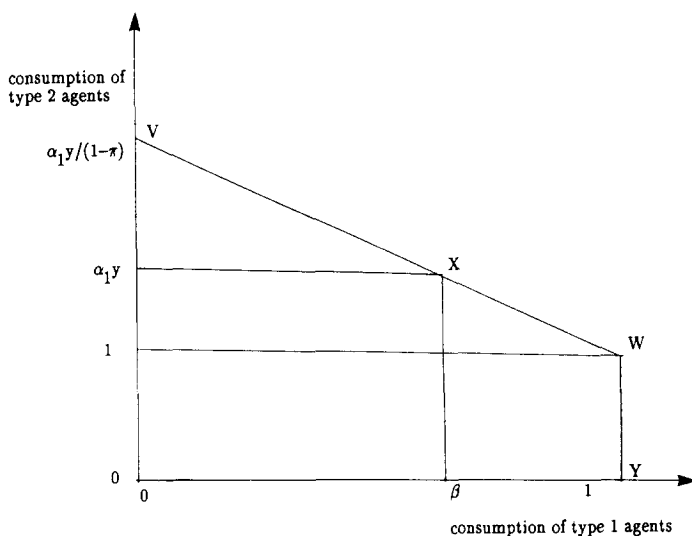


FIG. 4. Monetary SLE with circulating notes (Case M2).

notes are held because of their role as a medium of exchange, but the circumstances that make “liquid” assets useful also create an incentive for agents to issue bad circulating notes to sell on the secondhand note market. From (37), the existence of the Gresham’s law equilibrium is more likely (in the sense that the left side of the inequality is decreasing in the relevant parameter), and the existence of the equilibrium with fraud less likely, the smaller are β and α_1 and the larger are α_2 and π . Note also that, from (44), in the equilibrium with fraud the fraction of circulating notes traded which are good is increasing in β and α_1 , and decreasing in α_2 and π . These results are due to the fact that an increase in β decreases the incentive to issue a bad circulating note, an increase in α_1 (α_2) increases the two-period return on a good (bad) circulating note, and an increase in π increases the demand for liquid assets, which increases the incentive to issue a bad circulating note.

Any SLE has the property that circulating media of exchange are dominated in rate-of-return. That is, circulating claims coexist with bank deposits (noncirculating notes), which bear a higher two-period rate of return. In a nonmonetary SLE, transactions velocity and two-period rates of return are inversely related. Bank deposits have a transactions velocity of zero, a fraction π of the stock of good circulating notes changes hands each period, and the entire stock of bad circulating notes is traded in a period.

Rolnick and Weber (1983, 1984) have argued that, during the U.S. free banking era (1837–1863), which was essentially a *laissez-faire* regime,

fraudulent behavior in the issue of free bank notes was insignificant. The predictions of the model are consistent with Rolnick and Weber's reinterpretation of the U.S. free banking era in that there is no fraud in the Gresham's law equilibrium. That is, all circulating notes are bad and no agent can credibly claim otherwise. The Gresham's law equilibrium is more likely to exist if β , the cost of issuing a bad circulating note, is small. Interpreting the free banking era environment in terms of the model, it seems plausible to argue that β was quite low during this historical regime because of the plethora of bank notes in existence, which made the verification of bank note quality costly for individuals. Thus, the free banking environment would be more likely to produce a Gresham's law equilibrium, consistent with Rolnick and Weber's conclusions, than an equilibrium with fraud.

6. NONOPTIMALITY OF THE SLE

Consider a $(\bar{\cdot})$ allocation, defined as

$$x_t(t-1, 1) = x_t(t-2, 2) = y - \bar{k}, \quad (46)$$

$$z_{t+1}(t, 1) = 0, \quad (47)$$

$$z_{t+2}(t, 2) = \alpha_1 \bar{k} / (1 - \pi), \quad (48)$$

$$k_t^g = \bar{k}, \quad (49)$$

$$k_t^b = 0, \quad (50)$$

for $t = 1, 2, 3, \dots$, where $\bar{k} > 0$ solves

$$\max_k \{ \pi u(y - k) + (1 - \pi)v(y - k + \alpha_1 k / (1 - \pi)) \}. \quad (51)$$

By construction, the $(\bar{\cdot})$ allocation satisfies (1) and (2), and the expected utility of a young agent born at t given the $(\bar{\cdot})$ allocation is

$$\bar{U}_t = \pi u(y - \bar{k}) + (1 - \pi)v(y - \bar{k} + \alpha_1 \bar{k} / (1 - \pi)).$$

The incentive constraint (4) is satisfied, since (46) gives equal consumption allocations to middle-aged type 1 agents and old-aged type 2 agents at location 0. The participation constraint (5) holds since agents consume zero if type 1 in autarky, and positive amounts if either type in the $(\bar{\cdot})$ allocation. Therefore, the $(\bar{\cdot})$ allocation is feasible and incentive compatible.

PROPOSITION 4. *The SLE allocation is not Pareto optimal if the SLE exists.*

Proof. First, consider the case where (39) holds as a strict inequality. Therefore, from Propositions 2 and 3, a nonmonetary SLE exists and a monetary SLE does not. Let a $(\cdot)$ denote the SLE allocation. Agents who are old and middle-aged at $t = 1$ are better off with the $(\cdot)$ allocation, since

$$\begin{aligned}\hat{x}_1(-1, 2) &= \hat{x}_1(0, 1) = \hat{x}_2(0, 2) = 0 < \bar{x}_1(-1, 2) \\ &= \bar{x}_1(0, 1) = \bar{x}_2(0, 2) = y - \bar{k}.\end{aligned}$$

For the $(\cdot)$ allocation, let $\hat{k}$ denote the quantity of good trees that are produced each period to back noncirculating notes, so that $y - \hat{k}$ is the quantity of the consumption good exchanged by each young agent for newly issued and secondhand circulating notes. The expected utility of each agent born at $t = 1, 2, 3, \dots$, with the $(\cdot)$ allocation, is then

$$\begin{aligned}\pi u[(\bar{\alpha}/\beta)^{1/2}(y - \hat{k})] &+ (1 - \pi)v[(\bar{\alpha}/\beta)(y - \hat{k}) + \alpha_1\hat{k}/(1 - \pi)] \\ &< \pi u(y - \hat{k}) + (1 - \pi)v[y - \hat{k} + \alpha_1\hat{k}/(1 - \pi)] \\ &\leq \pi u(y - \tilde{k}) + (1 - \pi)v[y - \tilde{k} + \alpha_1\tilde{k}/(1 - \pi)].\end{aligned}$$

The first inequality follows from the fact that $u(\cdot)$ and $v(\cdot)$ are increasing and because $\bar{\alpha}/\beta < 1$ if (39) holds with strict inequality. Since $\tilde{k}$ is chosen to solve (51), we get the second inequality. Therefore, agents born at $t = 1, 2, 3, \dots$, strictly prefer the $(\cdot)$ allocation to the $(\cdot)$ allocation, and the SLE is not Pareto optimal in this case.

Next, consider the case where (39) holds with equality. Here, there exists a nonmonetary SLE and monetary SLEs, from Propositions 2 and 3. Agents born in periods $t = 1, 2, 3, \dots$, are indifferent between the $(\cdot)$ allocation and the SLE allocation. In the nonmonetary SLE, agents who are old and middle-aged at $t = 1$ consume zero and are therefore strictly better off with the $(\cdot)$ allocation. In the monetary SLE, from the proof of Proposition 3 in the Appendix, the price of fiat money, p , is such that the initial old and middle-aged are strictly better off with the $(\cdot)$ allocation. Q.E.D.

7. RESTRICTIONS ON PRIVATE NOTE ISSUE

We now want to ask whether imposing legal restrictions on some private claims could bring about a welfare improvement in the model with uncertain-lived agents. In this model, in contrast to the model with two-period-lived agents in Section 2, legal restrictions need not prohibit all intermediary liabilities, as there is more than one type of private claim in existence. Suppose then that the government prohibits all private circulating notes; such restrictions are very common in real-world fiat money

regimes. In our model, this restriction is possible because of the government's ability to monitor trading at location 0. We let the set of traded securities be otherwise identical to that in the previous section. A young agent born at time t then chooses fiat money balances m_t and noncirculating note acquisitions d_t to solve

$$\max[\pi u(c'_{t+1}) + (1 - \pi)v(c'_{t+2})] \quad (52)$$

subject to

$$0 \leq y - p_t m_t - d_t, \quad (53)$$

$$c'_{t+1} \leq p_{t+1} m_t, \quad (54)$$

$$c'_{t+2} \leq p_{t+2} m_t + [\alpha_1/(1 - \pi)]d_t, \quad (55)$$

$$m_t \geq 0, \quad d_t \geq 0. \quad (56)$$

The first-order conditions for an optimum, provided that $p_t > 0$ for all t , are

$$-\lambda_1 + \pi p_{t+1} u'/p_t + (1 - \pi) p_{t+2} v'/p_t = 0, \quad (57)$$

$$-\lambda_1 + \alpha_1 v' \leq 0, \quad (58)$$

where λ_1 is the Lagrange multiplier associated with the budget constraint of the young, and (58) holds with equality if $d_t > 0$.

In a stationary monetary equilibrium with legal restrictions (SME), $p_t = p > 0$ for all t , so that $p_{t+1}/p_t = p_{t+2}/p_t = 1$ for all t . There exists a unique SME and the SME allocation is the (") allocation of the previous section. Also, note that bank deposits (noncirculating notes) are held in the SME, by an argument identical to that for the monetary SLE when (45) holds.

Since the SME allocation is the (") allocation, it is immediate from Proposition 4 that this allocation Pareto dominates the SLE allocation if the SLE exists. It is also the case that the SME allocation is Pareto optimal, at least within a certain class of allocations.

DEFINITION 4. A *production stationary (PS) allocation* is an allocation where $x_t(t-1, 1) = x_2$, $x_t(t-2, 2) = x_3$, $z_{t+1}(t, 1) = z_2$, $z_{t+2}(t, 2) = z_3$, $k_t^g = k^g$, $k_t^b = k^b$, $t = 1, 2, 3, \dots$, where $x_i \geq 0$, $z_j \geq 0$, $k^g \geq 0$, $k^b \geq 0$, $i, j = 2, 3$.

PROPOSITION 5. *The SME allocation with legal restrictions is Pareto optimal within the class of PS allocations.*

Proof. As there are not fruit trees in existence at $t = 1$, we have $z_2(1, 1) = 0$. Therefore, a PS allocation has $z_2 = 0$, $z_3 = \alpha_1 k^g + \alpha_2 k^b$. Incentive compatibility, from (4), implies that $x_2 = x_3$. Therefore, since the (")

allocation is a PS allocation, the $(\bar{\cdot})$ allocation is the unique incentive compatible allocation in the set of PS allocations that maximizes the expected utility of agents born at $t = 1, 2, 3, \dots$. Therefore, there is no other incentive-compatible allocation in the set of PS allocations that could make at least one agent better off without making another worse off. Since the $(\bar{\cdot})$ allocation is incentive compatible and feasible and the SME allocation and the $(\bar{\cdot})$ allocation are identical, the SME allocation is Pareto optimal in the set of PS allocations. Q.E.D.

Legal restrictions banning circulating notes assure that a Pareto optimal SME always exists and that this SME Pareto dominates any SLE that exists.³ Note that it is important for this result that fruit trees disintegrate after bearing fruit. After the tree dies, a claim to the fruit from the tree can be counterfeited, and therefore becomes worthless. If this were not the case, then circulating notes could continue to circulate after the underlying asset paid off. Then, under some circumstances, a SLE supporting the $(\bar{\cdot})$ allocation exists, where circulating notes which have already paid off are valued.⁴ Thus, it is critical for our results that the technology is such that fiat money cannot be produced by the private sector, while claims to dead trees can be produced costlessly.

There are several features of the SME that are consistent with what is observed in real-world fiat money regimes. In particular, valued fiat money coexists with bank deposits (noncirculating notes), and the assets backing bank deposits dominate fiat money in rate-of-return. Also, fiat money has a higher transaction velocity than does the alternative asset.

As in the private information model of Section 2, a stationary equilibrium with valued fiat money exists with legal restrictions, where such an equilibrium does not exist without these restrictions, and the imposition of legal restrictions brings about a welfare improvement. The SME allocation is Pareto optimal while the SLE allocation is not, and the first allocation Pareto dominates the latter. These results can be contrasted to Sargent and Wallace (1982), where restrictions on private intermediation make some agents better off and some worse off, and where a Pareto optimal equilibrium always exists under *laissez faire*. Although Sargent and Wallace's model is somewhat different from ours (there is no private information in their model and they have borrowing and lending within each generation), they deal with a similar issue: the optimality of legal restrictions on money substitutes.

With legal restrictions, the fact that fiat money is recognizable to all and cannot be privately produced makes it an efficient medium of exchange

³ Note that, with legal restrictions, there is another stationary equilibrium where fiat money is not valued, i.e., where $p = 0$. This equilibrium is Pareto dominated by the SLE if the SLE exists.

⁴ Thanks go to a referee for pointing this out.

for use in trading at location 0. Private banking at locations $t = 1, 2, 3, \dots$, provides the incentive structure for efficient production to take place. Government intervention here is minimal, since the government need only prohibit certain circulating claims. In the absence of such intervention, private agents have the incentive to issue circulating claims which substitute for the government's circulating medium of exchange. As a result, bad private circulating claims drive out the government-provided medium of exchange and partially or completely drive out good quality private media of exchange.

8. SUMMARY AND CONCLUSIONS

The model constructed here is consistent with the conventional view that the provision of circulating media of exchange should not be left to the private sector. The model delivers this result while yielding predictions consistent with what is observed in free banking regimes and in fiat money regimes.

In a stationary laissez-faire banking equilibrium, fiat money is not valued (except in a hairline case), and the resulting allocation is not Pareto optimal. However, if the government bans the issue of private circulating notes, there exists a stationary equilibrium with valued fiat money that Pareto dominates the laissez-faire equilibrium and is Pareto optimal (within a certain class of allocations). These results stem from features of the model intended to capture the nature of monetary exchange. In particular, in some trading situations agents are unable to distinguish perfectly among private claims of varying quality. With no restrictions on private financial arrangements, private agents can exploit this informational asymmetry by issuing circulating notes backed by inferior assets, and a lemons problem results. Thus, there are advantages to having the government be a monopoly supplier of a universally recognizable medium of exchange.

In Williamson (1989), I argued that the banking regime in place in Canada during the period 1870–1913 worked quite well, and similar claims are made in White (1984) for the Scottish system of the early nineteenth century. Since both systems permitted the issue of private bank notes, is this historical experience consistent with the view that legal restrictions on financial intermediaries are optimal? I argue yes, since there were important legal restrictions in place in the Canadian and Scottish systems that played a role similar to restrictions on circulating notes. Principal among these restrictions were legal barriers to entry in banking, which made the Canadian and Scottish systems differ sharply from the regime in place in the United States during the free banking era. The existence of these barriers to entry was reflected in branch banking systems in Canada

and Scotland, with very few banks relative to the numbers that proliferated in the United States (Williamson, 1989; White 1984). If there are fewer banks, and banks are large, then information on the quality of bank portfolios should be more widely disseminated. Thus, if it can be made more costly to enter the business of banking, and if potential entrants are carefully screened, there is an alternative means for mitigating or eliminating potential lemons problems in the market for private circulating claims. An interesting area for future research is the interaction among various legal restrictions, including the prohibition of circulating notes, legal barriers to entry, and branch banking restrictions, and the historical performance of banking systems under these different restrictions.

APPENDIX

Proof of Proposition 2

We first prove that a Case N1 SLE does not exist. Suppose that it does. Then, the supplies of newly issued and secondhand circulating notes are zero. Since $p = 0$, an agent can consume in middle age (if type 1) only by acquiring a newly issued circulating note when young and selling it in middle age, or acquiring a secondhand circulating note when young and redeeming it in middle age. If $q^g = 0$ or $q^b = 0$, then there is excess demand for good or bad secondhand circulating notes (given the positive demand by young agents) and if $q^g > 0$ or $q^b > 0$, then there is excess demand for either good or bad newly issued circulating notes (given the positive demand for such notes by young agents). We therefore have a contradiction.

Next, suppose that a Case N2 SLE exists. Then (34) holds with equality, that is, using (40) and (41) to substitute in (34),

$$-\lambda + \pi(\alpha_2/\beta)^{1/2}u'[y(\alpha_2/\beta)^{1/2}] + (1 - \pi)(\alpha_2/\beta)v'(y\alpha_2/\beta) = 0. \quad (A1)$$

Then from (A1), the left side of (35) is equal to

$$\begin{aligned} & -\pi(\alpha_2/\beta)^{1/2}u'[y(\alpha_2/\beta)^{1/2}] + [\alpha_1 - (1 - \pi)(\alpha_2/\beta)]v'[y(\alpha_2/\beta)] \\ & > \pi\{-(\alpha_2/\beta)^{1/2}u'[y(\alpha_2/\beta)^{1/2}] + \alpha_1v'(\alpha_1y)\} \\ & \geq 0, \end{aligned}$$

so that (35) does not hold, a contradiction. The first inequality follows from $\alpha_1 > (\alpha_2/\beta)^{1/2}$ and the concavity of $v(\cdot)$, and the second inequality is implied by (36).

We next prove that a Case N3 SLE exists if and only if (37) holds. To prove necessity, if a Case N3 SLE exists, then (34) and (35) hold with

equality. Therefore, substituting in (33) using (34) and (35), (33) holds if and only if (37) holds. To prove sufficiency, suppose that (37) holds. Then, there is a unique $\tau > 0$, where $0 < \tau < 1$, such that, if agents face prices $p = 0$ and $q = (\alpha_2\beta)^{1/2}$, then τ is the optimal fraction of the endowment of the young saved in the form of bad circulating notes and second-hand circulating notes, and $1 - \tau$ is the optimal fraction of their endowment saved in the form of noncirculating notes. Here, $c_2 = \tau y(\alpha_2/\beta)^{1/2}$ and $c_3 = \tau y(\alpha_2/\beta) + (1 - \tau)y\alpha_1/(1 - \pi)$. Let k_t^b denote the quantity of bad fruit trees planted in period t . Then it is straightforward to show that there exists a unique sequence of nonnegative equilibrium asset quantities $\{k_t^b\}$, which is the solution to a linear difference equation with initial condition $k_1^b = \tau y/\beta$.

Next, suppose that a Case N4 SLE exists. Then (42) holds, and the expected utility of a young agent born in any period is

$$\pi u[\tau y(\bar{\alpha}/\beta)^{1/2} + (1 - \tau)y(\bar{\alpha}\beta)^{1/2}] + (1 - \pi)v[\tau y\bar{\alpha}/\beta + (1 - \tau)y\alpha_1], \quad (A2)$$

where τ is the fraction of savings invested in new bad circulating notes and secondhand circulating notes. Differentiating (A2) with respect to τ , we obtain

$$\begin{aligned} & y\{[(\bar{\alpha}/\beta)^{1/2} - (\bar{\alpha}\beta)^{1/2}]u'[\tau y(\bar{\alpha}/\beta)^{1/2} + (1 - \tau)y(\bar{\alpha}\beta)^{1/2}] \\ & + (1 - \pi)(\bar{\alpha}/\beta - \alpha_1)v'[\tau y\bar{\alpha}/\beta + (1 - \tau)y\alpha_1]\} \\ & \leq y\pi[(1 - \beta)/\beta]\{(\bar{\alpha}/\beta)^{1/2}u'[\tau y(\bar{\alpha}/\beta)^{1/2} + (1 - \tau)y(\bar{\alpha}\beta)^{1/2}] \\ & - \alpha_1v'[\tau y\bar{\alpha}/\beta + (1 - \tau)y\alpha_1]\} \\ & < y\pi[(1 - \beta)/\beta]\{(\bar{\alpha}/\beta)^{1/2}u'[y(\bar{\alpha}\beta)^{1/2}] - \alpha_1v'(y\alpha_1)\} \\ & \leq 0, \end{aligned}$$

for $0 \leq \tau \leq 1$. Here, the first inequality follows from (42), the second inequality is a consequence of the concavity of $u(\cdot)$ and $v(\cdot)$, $\beta < 1$, and $\bar{\alpha}/\beta < \alpha_1$, and the third inequality follows from (36). Thus, if agents are constrained to holding a convex combination of good circulating notes and bad circulating notes, they will choose a corner solution with $\tau = 0$. But, in a Case N4 SLE, agents choose an interior solution for τ ; therefore there is a contradiction.

Next, we prove that a Case N5 SLE exists if and only if (38) and (39) hold. In a Case N5 SLE, (33), (34), and (35) hold with equality. Substituting using (33) and (34) in (35), conditions (33), (34), and (35) hold with equality if and only if (43) holds. We also have (44). In a Case N5 SLE, $0 \leq \gamma^* < 1$, which holds if and only if (38) holds. In a Case N5 SLE, if a young agent chooses τ to maximize (A2) (that is, if a young agent could

save in the form of good and bad circulating notes only), she would not choose an interior solution for τ , from the previous arguments. Thus, given (43), agents will choose to hold some noncirculating notes.

An individual agent can achieve any allocation on her budget constraint in a Case N5 equilibrium by holding a convex combination of bad circulating notes and noncirculating notes. Therefore, the expected utility of a young agent can be written as

$$\pi u[\tau y(\bar{\alpha}/\beta)^{1/2}] + (1 - \pi)v[\tau y(\bar{\alpha}/\beta) + (1 - \tau)y\alpha_1/(1 - \pi)]. \quad (A3)$$

Let τ^* denote the value for τ that maximizes (A3). Letting k_t^b and k_t^g denote the quantities of bad and good circulating notes, respectively, issued in period t , we must have

$$\pi k_t^g/(\pi k_t^g + k_t^b) = \gamma^*, \quad (A4)$$

for all t . Now, note that the portfolio allocation τ^* can be attained if a young agent born in period t allocates a fraction η_t of her portfolio to noncirculating notes, a fraction $(1 - \eta_t)\delta_t$ to good circulating notes and the remaining fraction $(1 - \eta_t)(1 - \delta_t)$ to bad circulating notes and second-hand circulating notes. Here, $0 \leq \eta_t, \delta_t \leq 1$. This portfolio must attain the same returns as the τ^* portfolio. That is,

$$(1 - \eta_t)\delta_t(\bar{\alpha}/\beta)^{1/2} + (1 - \eta_t)(1 - \delta_t)(\bar{\alpha}/\beta)^{1/2} = \tau^*(\bar{\alpha}/\beta)^{1/2}, \quad (A5)$$

for all t . The stock of good fruit trees must be equal to the quantity of savings allocated to good circulating notes in each period, or

$$k_t^g = \delta_t(1 - \eta_t)y, \quad (A6)$$

for all t . Also, the market for secondhand circulating notes must clear. That is,

$$(1 - \eta_t)(1 - \delta_t)y + (1 - \pi)(\bar{\alpha}/\beta)^{1/2}(1 - \eta_{t-1})(1 - \delta_{t-1})y - \beta k_t^b \\ = (\bar{\alpha}/\beta)^{1/2}(k_{t-1}^b + \pi k_{t-1}^g), \quad t = 2, 3, 4, \dots, \quad (A7)$$

$$(1 - \eta_1)(1 - \delta_1)y - \beta k_1^b = 0. \quad (A8)$$

In (A7), the right side denotes the demand for secondhand circulating notes, and the left side denotes the supply, valued in terms of the period t consumption good. For $t = 1$, Eq. (A8) has a similar interpretation, where the supply of secondhand notes is zero.

An equilibrium then consists of a sequence $\{k_t^b, k_t^g, \delta_t, \eta_t\}_{t=1}^\infty$, which satisfies (A4)–(A8) and $k_t^g \geq 0, k_t^b \geq 0, 0 \leq \delta_t \leq 1, 0 \leq \eta_t \leq 1$ for all t . It is

straightforward to show that a unique equilibrium exists if $\bar{\alpha}/\beta \leq 1$. Therefore, from (43) and (44), (39) is a necessary condition for existence, and (38) and (39) together are necessary and sufficient. Q.E.D.

Proof of Proposition 3

We first prove that, in a monetary SLE, either good and bad circulating notes coexist or no circulating notes are issued. Suppose not. Then bad circulating notes are issued and good circulating notes are not. Therefore, we have (30) and $\gamma = 0$ which, from (41), gives $q = (\beta\alpha_2)^{1/2}$. Therefore, the one-period return on a new bad circulating note or a secondhand circulating note is $(\alpha_2/\beta)^{1/2} < 1$, and the two-period return from investing in either asset, selling it after one period, and investing the proceeds in new bad circulating notes or secondhand circulating notes is $\alpha_2/\beta < 1$. Therefore, fiat money dominates these other assets in rate of return, a contradiction.

A monetary SLE, if it exists, will then be one of the following cases, where A is now the set of assets held in equilibrium in addition to fiat money.

- Case M1. $A = \{G, B\}$
- Case M2. $A = \{G, B, D\}$
- Case M3. $A = \{D\}$
- Case M4. $A = \{\}$.

We next prove that, if $\alpha_1(\beta - \pi) - \beta(1 - \pi) \neq 0$, then a monetary SLE does not exist. Using arguments similar to those in Proposition 5, it can be shown that a Case M1 equilibrium does not exist. That is, given the asset returns faced by agents in Case M1, agents would choose not to hold bad circulating notes or secondhand circulating notes, a contradiction.

In Case M2, as in the Case N5 equilibrium, (43) must hold. However, it must also be the case that the returns on bad circulating notes, secondhand circulating notes, and fiat money are identical or $\bar{\alpha}/\beta = 1$ and $\gamma = (\beta - \alpha_2)/(\alpha_1 - \alpha_2)$. But if $\alpha_1(\beta - \pi) - \beta(1 - \pi) \neq 0$, then $\bar{\alpha}/\beta \neq 1$ and the equilibrium does not exist.

Next, for Cases M3 and M4, circulating notes are not issued. Here, we have (31). The absence of the rate-of-return dominance implies that the rate of return on fiat money must be greater than or equal to the rate of return on a secondhand good note, i.e., $\alpha_1/q^g \leq 1$. Also, a good circulating note must not dominate fiat money if it were issued. That is, $q^g \leq 1$. But, since $\alpha_1 > 1$, both inequalities cannot hold. Therefore, we have a contradiction.

Next, we prove that, if $\alpha_1(\beta - \pi) - \beta(1 - \pi) = 0$, then a continuum of monetary SLEs exist, indexed by $p \in (0, \tau^*y/H)$, where τ^* is the value of τ that maximizes (A3). If a monetary SLE exists, then it must be a Case M2 SLE, from the proof of Proposition 8. In a Case M2 SLE, $\bar{\alpha}/\beta = 1$ and

$\gamma = (\beta - \alpha_2)/(\alpha_1 - \alpha_2) \equiv \gamma^*$. The analysis is then similar to that for the Case N5 SLE in the proof of Proposition 6. Let τ^* be the choice for τ that maximizes (A3), where τ^* is the fraction of the savings of a young agent invested in secondhand circulating notes and newly issued bad circulating notes, when agents have the option of holding a convex combination of noncirculating notes, secondhand circulating notes, and newly issued bad circulating notes. Let η_t , μ_t , $(1 - \eta_t - \mu_t)\delta_t$, and $(1 - \eta_t - \mu_t)(1 - \delta_t)$ denote the fractions of the savings of young agents in period t invested in noncirculating notes, fiat money, good circulating notes, and in bad circulating notes and secondhand circulating notes, respectively. Then, similar to Case N5, we can construct a monetary SLE which is a sequence $\{k_t^b, k_t^g, \eta_t, \mu_t, \delta_t, \phi_t\}_{t=1}^\infty$. Here, $\phi_1 = pH/y$, and p is indeterminate, with $p \in (0, \tau^*y/H)$. Q.E.D.

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